A Journey with Number

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Abstract

Number and Numeracy have come under the spotlight in education in New Zealand over the last decade. This article is a historical introduction to the use and learning of Number, as a quantity and as a quality, and offers a personal perspective of the Western World's cultural impact on the teaching of Numeracy in New Zealand[1]. I have found in my work with students who have difficulties with Numeracy and Mathematics that the thresholds these students face often mirror thresholds found in the historical use of Number. And just as history shows that there has had to be a 'jump' in conscious understanding across those thresholds, so too has the modern-day pupil needed to make such 'jumps'.

A Beginning

We begin where agricultural farming is believed to have originated along the Tigris, Euphrates and Nile valleys. During these Sumerian-Egyptian times the priest-scribes of the temples possessed the formulae within which they 'put' numbers to get their tallies (Bowen, 1972). These formulae and the ability to use number-tallies were considered to be of divine origin, hence their use only by temple priest-scribes. Storage of harvested crops was managed communally within the temples' precincts, and priests gave the growers tokens according to the amount tallied by the priest-scribes, to be redeemed as required during the 'lean' months. These clay tokens were also a precursor to money, and the number inscribed described the quantity of harvested goods stored in the temples' precincts according to the formulae that the priests used.

Arithmetic, as practised during the twentieth century in New Zealand schools up until the early-mid 1970s, often encouraged a similar 'use' of formulae (Campbell, 1941; MoE, 1992). Teachers (and the textbooks) would present the formulae and show how to find 'the answer' by inserting figures. If one could memorise the formulae and know which figures to put where, the 'right answer' could be (miraculously!) calculated. If one couldn't, the process to a 'right answer' seemed totally out of reach; individual pupils thus felt themselves to be 'not good' enough.

Also during Assyrian-Egyptian times geometric measurement, according to the 'normal' length of a finger digit, foot, hand, forearm, was used to quantify space (Wilkinson, 1978). Buildings (and pyramids!), roads and canals were all developed according to the area and volume measurement calculations of the priest-scribes. Clay tablets and papyri show 'tables' used for quick reckoning in multiplication and division. The Egyptian-Sumerian number system was base-10, but the Babylonians used a base-60 number system, which was later used almost exclusively by astronomers. We still have minutes/seconds to measure angles and time, and use 'hands' to measure the height of a horse.

The Greek Impulse

A qualitative view of number didn't appear until the Greek times of early philosophers such as Pythagoras (570-495BCE) and Protagoras (c.485-411BCE). The Pythagorean School, centred in what is now southern Italy (c.530-500BCE), promulgated the idea that music, arithmetic, geometry and astronomy made up mathematics (from Greek: *máthēma*, the 'study of learning, knowledge'). Pythagoras, through the study of harmonics and music, recognised proportion and the relationship (through sound) between numbers; Arithmetic considered the

1 Although New Zealand enjoys a place in the Pacific, the teaching/learning tradition has been based on Western cultural experiences, rather than Eastern or Oceanic, and I have therefore focused on the historical conventions from that 'Western World'. The 'discrepancy' between this Western model and many Māori and Pasifika students' learning may indicate a reason for the difficulties experienced by the students. (Cf Bishop (1995), Western mathematics: The secret weapon of cultural imperialism.)

'nature' of the number (Greek: *arithmos*), its qualities of one-ness, two-ness, etc., and its virtues (perfect, amicable); Geometry extended ratio and proportion derived from harmony to include two-dimensional and three-dimensional figures; Astronomy included the application of proportion, harmony, and geometry to celestial relationships.

Protagoras, a Sophist and Rhetorician, is credited with the following:

> *"Of all things the measure is man,*
> *of the things that are, [how] that they are,*
> *and of the things that are not,*
> *[how] that they are not".*[2]

Protagoras brought philosophy to a human level suggesting that knowledge is relative to the person 'doing' the knowing, i.e. thinking. Thought became its own independent faculty. Thus, a consideration of the quality of a number coincided with a more individual and conscious style of thinking; a change in consciousness that prefigured existentialism.

During the time of Plato (428-348BCE) and Aristotle (384-322BCE), grammar, dialectics (logic) and rhetoric were added to the four 'arts' that made up mathematics. A well-educated 'free person' needed to have studied all seven, and Plato's and Aristotle's schools encouraged periods of seven years or more to progress from grammar through dialectics, rhetoric, arithmetic, geometry, astronomy to the ultimate, music (Bowen, 1972). Boethius (480-524/5CE) suggested that Number 'in itself' is arithmetic, 'in relations' is music, as 'quantity at rest' is geometry, as 'quantity in motion' is astronomy (Wagner, 1983).

Liberal Arts and Practical Application

While these Liberal Arts (from the Latin: *artes liberalis* - subjects of study proper to free persons) were studied during Graeco-Roman times[3], the Dark Ages precluded much of their study in the West. The School of Chartres (11th - 12th centuries) re-established them[4], but it was not until after the Black Plague of the 14th century, when universities began to flourish[5], that the study of the Seven Liberal Arts including the philosophical study of the quality of number, became commonplace (Bowen, 1972; Querido, 2012; Wagner, 1983). Roger Bacon (1214-1294) and John of Holywood (d.1256) both advised that theoretical and practical mathematics be studied in universities.

Meanwhile, the practical application of geometry and arithmetic (Greek: *logistiké* - processes of calculation; Latin: *computāre* - to deem or think, to count or reckon) continued. Surveying of land areas and for roads, mechanics and engineering for construction, and the use of abacus and tallies in the market place ensured that formulae and number as a quantity were constantly practised. Roman numerals were still the written form of numbers, and the clumsiness of the forms prevented much of the more advanced arithmetic and algebra enjoyed today. Gerbert of Aurillac (c.955-1003CE) is believed to have introduced the decimal system and, as he had studied in Spain, to have also used Arabic numerals in his own calculations.

'New' Numbers

In the early 13th century, Fibonacci, who had travelled through the Middle East and Moslem dominions (from Spain to the borders of China) with his merchant father, translated Arabic texts and introduced the Indo-Arabic numeral forms to a wider audience in his book "Liber Abaci". Initially thought of as black magic because their symbols hid common understanding, the numbers were not widely used until the 15th century (Kaplan, 1999).

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2 According to Plato's writings Theaetetus. This quotation is not known in what survives of Protagoras's written works.

3 Note also Euclid (fl. 300BCE) and Archimedes (c.287-c.212BCE) and their contribution to mathematics and especially geometry.

4 See descriptions and photographs of the carved Western Portal of Chartres Cathedral, where the seven liberal arts, personified as feminine beings, are depicted within the right hemispherical tympanum. Geometrica and Arithmetica stand at the top, with Astronomica, then Musica and Grammatica on one side, and Rhetorica and Dialectica, with Gemini on the other side of the hemisphere.

5 Universities in Paris and Oxford (Merton College) began in the early thirteenth century.

Basic computation (along with some reading and writing of letters - the three 'Rs') had been offered to those who attended small schools associated with cathedrals and local parishes, from the Middle Ages on. In trade and commerce, the abacus, tally sticks and sand tallies, finger counting and knotted strings persisted as counting forms into the Industrial Age, when charity/industrial, as well as the church/parish schools began to give a wider group of the population a chance for some education - and social control (Bishop, 1995; Bowen, 1972; Campbell, 1941). Up until then, only those who could afford tutors or governesses, and university education, were able to take advantage of the 'purer' aspects of mathematics and number.

Finger/digit counting and tallies are still the most common early forms of counting used by pupils today. This concrete representation seems to help consolidate the learning of quantity and magnitude, and there are those students with dyscalculia who cannot easily advance to the more abstract Indo-Arabic numbers that symbolise quantity, yet manage very well with Roman numerals (Rousselle & Noël, 2006). Tamsin Meaney (2006) also rightly considers acquiring a mathematics register as a 'second language' in the classroom environment.

Mathematics Education in New Zealand

In New Zealand, the 1877 Education Act allowed for free, secular and compulsory primary education for up to six years, for children six years of age and over. Up until then, all schools had been based on the English 19th century system of church/charity schooling. The Act also allowed for some provision of secondary schooling through district high schools. Separate acts of Parliament established high schools, which were not free, and followed 'classical' curricula and the traditions of English public and grammar schools (Fraser, 1986). The 'classical' mathematics curriculum included and prepared the student for the more philosophical approach (aligned to the 'seven liberal arts') still practised in universities. In 1904, George Hogben, as Inspector-General of Education, introduced a syllabus aiming for character formation with a moralistic base (Fraser, 1986) that extended and formalised the 19th century social control approach used in teaching.

It was not until the late 1960s - early 1970s that a child-centred (rather than moralistic society-centred) approach to learning developed out of the neo-romantic radicalism of the time. 'Manipulables' were introduced into basic mathematics (arithmetic) to help the pupil move from finger counting, and to objectify the learning of computation. During the 1980s an economic paradigm of 'input' and 'output' arising from the Scientific Management Theory began to infiltrate the classroom and, in particular, mathematics (Neyland, 2002).

The 1992 (MoE, Mathematics in the New Zealand Curriculum) and 2007 (MoE, The New Zealand Curriculum) curriculum publications encourage a broad learning for a healthy integration of the individual into society. The Numeracy projects (NDP, from 1999), an important part of the government's Literacy and Numeracy strategy up until 2010, focus mathematics learning on discrete steps of numeracy acquisition (Thomas, Tagg & Ward, 2002; MoE, 2005-2007). Beginning in the first year at school with 5 year-olds, the NDP continues for ten years before the National Certificate in Educational Achievement (NCEA) programmes follow through years 11, 12 and 13. While there is a deserved focus on the pedagogical content knowledge (PCK) of the teacher, the concentration on achievement of learning steps in numeracy shrinks the enjoyment of mathematics to basic arithmetic and algebra. The recent law regarding National Standards of numeracy tightens this shrinkage.

Unfortunately, the number of changes made to the method of mathematics education in New Zealand schools since the 1970s could be contributing to the lower achievement levels now seen among the 9-11 year olds in international tests. Schuberth (1999) specifically points to changes in methods of teaching, especially when abrupt, contributing towards dyscalculia in pupils.

Conclusion

It could be argued that a thorough understanding of the basic concepts of numeracy is rudimentary to the broader mathematics curriculum. While students continue to complete their schooling without such a numeracy understanding, there is concern for their future contribution to society. Education is considered an important aspect of the continuing culture of a societal group (Bowen, 1972), but education for the good of society leaves out the importance of education for the individual. A philosophical context, that encompasses existentialism, will do more for the growth and adaptability of a society and culture, than an orientation on discrete steps of acquisition. The quality of a number is just as important as the quantity. Too often, we lose sight of the 'wood for all the trees'. And, just as in nature, students may feel lost within the concentration on the details. Historically, the practice of arithmetical computation within technology (abacus and tally boards) helped anchor such learning, and may now give valued and real context to numeracy acquisition. The concept of Number, as presently taught within New Zealand schools aiming to meet the National Standards, has shrunk to a social need rather than being a learned element of a healthily integrated person. ◆

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