From beauty to truth in mathematics

Affluence is a poor spur

By R. A. JARMAN

“TWINKLE, twinkle, little star, how I wonder what you are.” Trite, but the search for truth in any branch of knowledge always begins with a sense of wonder. Nowhere is this more evident than in the child and the teacher counts upon it as his, greatest helper.

But wonder dawns moot brightly in the presence of the beautiful, for here there is immediate awareness of a harmony, created, one often feels, by considerable inner effort on the part of some being or beings, and whose source lies deeper than physical or psychological levels.

To anyone willing to engage in mathematical activity and then gradually to penetrate to its foundations a sublime beauty is revealed. This is no less true of the so-called “modem mathematics” than of the more classical kind. It is unfortunate that the introductory work presented in many modem textbooks is little’ short of treason to the true spirit of mathematics.

There is an over-anxiety to plunge straight into technical applications. So set theory can appear as a by-product of public opinion polls. The majesty of geometrical transformations can so easily be lost by ignoring the movements which give birth to them and attending solely to the correspondence and its symbolic expression between two or more configurations. It can be likened to glancing through the frames of a strip cartoon, spotting the connections, but ignoring what is happening between the events.

Modem mathematics can also strike pupils merely as a game to be played according to precise rules, sharpening the intellect and training it in strategy and wilefulness, but otherwise only obliquely related to what really matters in life and having nothing to say to their hearts. The chief spur to the pupil is so often the prospect of passing an exam which will open the door leading to the spoils of our affluent society.

Treason, stratagems and spoils always come on the scene in consequence of the lack Shakespeare so tellingly observed. For it is equally true that no one can know or be active within the real spirit of mathematics who has no music in his soul. Regular participation in choral singing or instrumental music groups ought to be obligatory in the training of a mathematics teacher. Not that this is the music in soul which is meant, but it is closely related and is a powerful stimulant

Just as with other subjects, the first step when introducing a new topic in mathematics to a class should be the stimulation of wonder for the pure phenomenon. With the youngest children the teacher can discuss where “oneness”, “two ness” and “three ness” are in the world. “Two ness”, something itself invisible, appears in connection with ears, hands, parents; “oneness” with the sun, the heart; “seven ness” with the days of the week, the rainbow, and so on, and children can expand such examples to great lengths.

The feeling of enthusiasm generated by wonder can be led into practical activity. The will of the rhythmically active child, especially in his hands, brings to him the experience of the phenomenon, be it in counting, multiplication tables, solving equations or geometrical constructions. The third step is the discovery of a new truth. The thought activity is awakened by the will. The pupil's discovery may, for instance, be that of the commutative law-three fives make the same as five threes-or of a geometrical proposition.

Let us take geometrical examples first. With the introduction of compasses to ten-year-old children, wonder is stimulated by drawing the seven-circle pattern shown in figure I. No matter where you start, it always works-the same radius for all seven, three circles each time through six of the centres and six circles through the middle one. The teacher's presentation can stimulate the children to draw the figure again and again in different sizes and then elaborate and develop it with more circles. Then the children themselves will discover that the secrets of right angles, equilateral triangles and hexagons, the bisection of angles and so on are all found from contemplating the results of their drawing activity.

Later on the laws of the limacon-cardioide family can be discovered by a further elaboration and metamorphis of this figure. The cardioide envelope (figure II) is formed by moving circle whose centre moves on a fixed circle but whose circumference still passes through a fixed point which is now on the fixed circle. It is the movement which is all essential.

All form is frozen movement. Whenever thought discovers something new, it is through stepping out of a movement in which we are ourselves engaged, bringing it momentarily to rest and perceiving the arrested form.

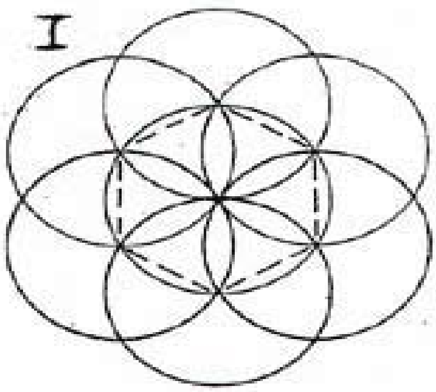
Yet another metamorphis, i.e. change of form-producing movement, of figure I is seen in figure III, drawn quite independently of any measurement. It can be used as a basis of projective geometry for older boys and girls in exactly the same way as figure I for tenyear-olds. The hexagon here is a shadow thrown from the perfect hexagon of figure I, yet its construction is simple and independent of the shadow idea.

Applying mathematics is a fourth step. The truths won from the third (contemplative) step can be put to service but it is a mistake always to want to apply knowledge immediately to technology. The history of mathematics demonstrates that almost all its important discoveries were made without any reference to their application in most cases applications were neither found nor even conceived until centuries later. Socrates' desire that the subject be pursued in the spirit of a philosopher rather than of a shopkeeper was not just aimed against human acquisitiveness. It is the only way in which mathematical progress can be made.

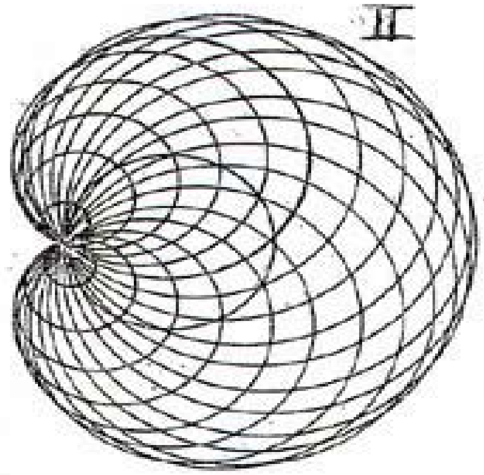
Nonetheless it is essential that the teacher show his pupils really practical applications of the mathematics taught, when the moment is ripe. The wonders of Pythagoras' theorem—all the drawing, cardboard-cutting and making of movable models with wires or string which lead to its recognition-are enhanced when it is applied to engineering structures or vector navigational problems. On the other hand, harm can be done by applying equations to situations of no practical worth when simple arithmetic can reach the answer more speedily anyway.

If no really practical problem is apposite at a certain stage (this is not entirely true even of simple equations) it is far better for the teacher to tell his pupils that the practical value will be evident when

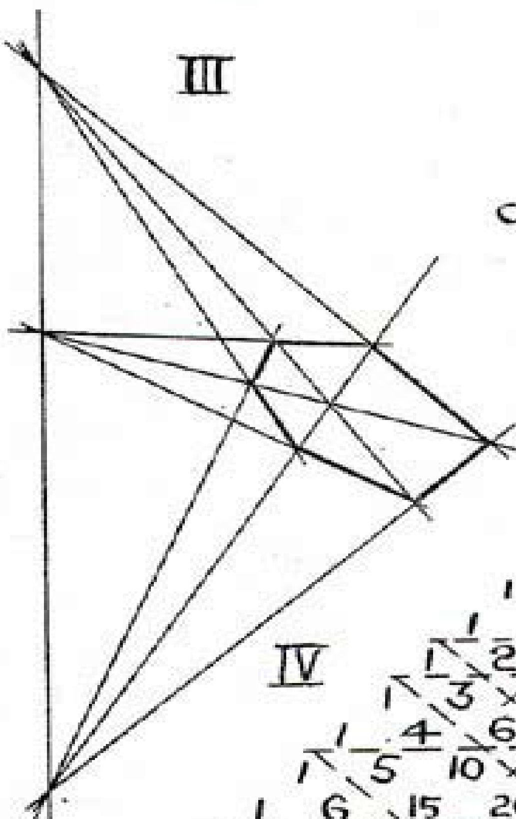
I



II



III



V

CUMULATIVE DIFFERENCES ↓

DIFFERENCES ↓

7 × 7 = 49		
	1	
8 × 6 = 48		1
	3	
9 × 5 = 45		4
	5	
10 × 4 = 40		9
	7	
11 × 3 = 33		16
	9	
12 × 2 = 24		25

IV

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1

they have developed algebra to a further level. Otherwise the radiance of the original wonder will be lost, like a cloud coming over the sun.

There is a more popular appreciation of beauty in geometrical *as* against arithmetical or algebraical form and metamorphosis. But the beauty to be found in the numerical is related to beauty in the measurable or extendible just as poetry is related to painting.

Fourteen-year-old boys and girls in a Steiner school have a main lesson period in permutations, combinations and the binomial theorem. A central event in this main lesson period is the construction and contemplation of Pascal's triangle, of which the first dozen rows are shown in figure IV. Apart from the repeated number ones in the sides of the pyramid every number is obtained by adding together the two numbers immediately above to the left and right of it.

Discoveries in abundance can be made by contemplating this army of numbers and the pupils' wonder grows ever stronger as they do so. For example, each dotted triangle contains a set of numbers having the property of being divisible by the number in its top corners—but this only works when such a number is prime.

From such a contemplation the whole theory of combination formulae and the binominal theorem develop.

It is what a Goethean would call a primary phenomenon. From the dotted triangles spring Venn diagrams and the intersection of sets.

Another array far younger children to contemplate is given in figure V. Again they become aware of that inner harmony which the beautiful ever demands. From it there can spring the truth that $(a + b)(a - b) = a^2 - b^2$.

In conclusion we may consider the question which the writer was asked to answer in this article. "In what ways do Steiner schools lead their students to find not only truth but beauty in mathematics?

The answer is that truth in mathematics—in its real, *most* spiritual and effective sense — may only be found *by* discovering beauty first. Wonder generates feelings of enthusiasm. This fires the will to do number work or geometrical construction. In turn this will activity awakens the thinking which finds the truths. In applying truths to the world in which we live the pupil who has been led consciously to them via beauty will not forget what he owes to the latter. He will endeavor to marry the beautiful to the technical in his work in the world.